$$\begin{cases} R(x{:}y) \text{ rational} \\ \text{no poles on } x^2+y^2=1 \end{cases} \Rightarrow \int_{dt/2\pi}^{0|2\pi} R(\cos t{:}\sin t) = \sum_{|z|<1} \operatorname{Res} \frac{1}{z} R\left(\frac{z+z^{-1}}{2}{:}\frac{z-z^{-1}}{2i}\right)$$

$$z = e^{it} \Rightarrow \begin{cases} dz = izdt \\ \cos t = \dfrac{z+z^{-1}}{2} \\ \sin t = \dfrac{z-z^{-1}}{2i} \end{cases} \Rightarrow \text{LHS} = \frac{1}{2\pi i} \int^{|z|=1} \frac{dz}{z} R\left(\frac{z+z^{-1}}{2}{:}\frac{z-z^{-1}}{2i}\right) = \text{RHS}$$

$$\int_{dt/2\pi}^{0|2\pi} \begin{cases} \dfrac{1}{a+\sin t} = \operatorname{Res} \dfrac{2i}{z^2+2iaz-1} \underset{\text{Der}}{\overset{\text{Ev}}{=}} \dfrac{2i}{2z+2ia}\Big|^{z=}_{i\sqrt{a^2-1}-ia} = \dfrac{1}{\sqrt{a^2-1}} \\ \dfrac{1}{2-\sin t} = \dfrac{1}{\sqrt{3}} \end{cases}$$

$$\int_{dt/\pi}^{0|\pi} \underset{\text{ev}}{=} \int_{dt/2\pi}^{0|2\pi} \begin{cases} \dfrac{1}{a+\cos t} = \operatorname{Res} \dfrac{2}{z^2+2az+1} \underset{\text{Der}}{\overset{\text{Ev}}{=}} \dfrac{2}{2z+2a}\Big|^{z=}_{\pm\sqrt{a^2-1}-a} \overset{a\geq 1}{=} \dfrac{1}{\sqrt{a^2-1}} \\ \dfrac{1}{3+2\cos t} = \dfrac{1}{\sqrt{5}} \\ \dfrac{1}{2+\cos t} \\ \dfrac{1}{5-3\cos t} \end{cases}$$

$$\int_{2dt/\pi}^{0|\pi/2} = \int_{dt/\pi}^{0|\pi} = \int_{dt/2\pi}^{0|2\pi} \begin{cases} \dfrac{1}{a+\sin^2 t} \\ \dfrac{a}{a^2+\sin^2 t} = \dfrac{2a}{1+2a^2-\cos t} = \dfrac{1}{\sqrt{1+a^2}} \\ \dfrac{1}{1+\sin^2 t} = \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{2+\cos^2 t} = \dfrac{1}{\sqrt{6}} \end{cases}$$

$$\int_{dt/2\pi}^{0|2\pi} \frac{\sin t}{a+\sin t}$$

$$\int_{dt/2\pi}^{0|2\pi} \frac{\cos(2t)}{5-4\cos t} \underset{\cos^2 t-\sin^2 t}{\overset{\cos(2t)=}{=}} \operatorname{Res} \frac{z^4+1}{-4z^2(z-1/2)(z-2)} = -\frac{1}{4} \begin{cases} {}^{0}\text{Der} \dfrac{z^4+1}{(z-1/2)(z-2)} = \dfrac{5}{2} \\ {}^{1/2}\text{Ev} \dfrac{z^4+1}{(z-2)z^2} = -\dfrac{17}{6} \end{cases} = \frac{1}{12}$$

$$\int_{dt/2\pi}^{0|2\pi} e^{\cos t} \underbrace{\cos(nt) - \sin t}$$

$$\int_{dt/2\pi}^{0|2\pi} \frac{1}{1+a^2-2a\cos t} = \begin{cases} \dfrac{1}{1-a^2} & 0<a<1 \\ \dfrac{1}{a^2-1} & 1<a \end{cases}$$

$$\int_{dt/2\pi}^{0|2\pi} \begin{cases} \dfrac{1}{(a+\sin t)^2} = \operatorname{Res} \begin{cases} \dfrac{-4z}{(z^2+2iaz-1)^2} = \\ \dfrac{-4z}{(z-a_+)^2(z-a_-)^2} \end{cases} = {}^{a_+}\mathrm{Der}\dfrac{-4z}{(z-a_-)^2} = \dfrac{a}{(a^2-1)^{3/2}} \\ \dfrac{1}{(b+a\cos t)^2} = \dfrac{b}{\left(b^2-a^2\right)^{3/2}} \\ \dfrac{1}{(2-\sin t)^2} \\ \dfrac{1}{(1+a\cos t)^2} \end{cases}$$

$$\int_{2dt/\pi}^{0|\pi/2} \frac{1}{\left(a+\sin^2 t\right)^2} = \frac{2a+1}{2\left(a^2+a\right)^{3/2}}$$

$$\overline{a} < 1: \quad \int_{dt/\pi}^{0|\pi} \underset{\text{ev}}{\equiv} \int_{dt/2\pi}^{0|2\pi} \begin{cases} \dfrac{\cos(2t)}{1-2a\cos t+a^2} \\ \dfrac{\cos(nt)}{1-2a\cos t+a^2} = \operatorname{Res} \dfrac{z^n}{(z-a)(z-1/a)} \end{cases}$$

$$\int_{dt/2\pi}^{0|2\pi} \sin^{2k} t = \frac{(2k)!}{4^k (k!)^2}$$